

## **Enhancing Graph Neural Networks** with Geometric Structure Analysis

### Maysam Behmanesh

behmanesh@lix.Polytechnique.fr



POLYTECHNIQUE DE PARIS



Nov.  $25^{th} - 2024$ 

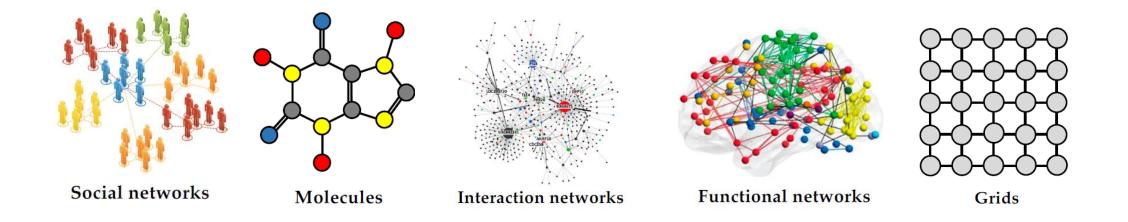
### Outlines

- 1. Machine learning on graphs
- 2. Graph Neural Networks (GNNs)
- 3. Challenges on GNNs
- 4. Innovative methods
- 5. Conclusions

### Motivation

### • Graphs are Everywhere!

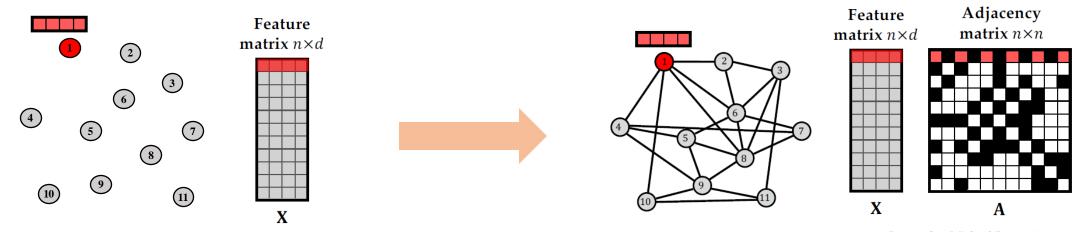
Growth of *diverse* graph based data: Social networks, Molecules, Interaction networks, Biomedical imaging ....



### Motivation

• Implicit graphs

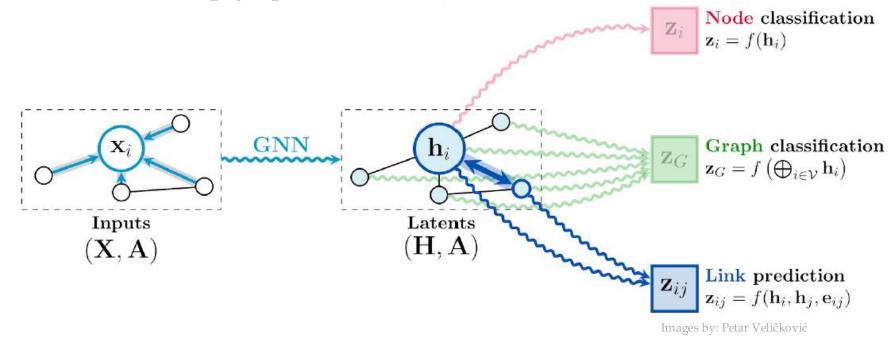
Inject geometric information into point cloud to form an *implicit graph* 



Images by: Michael Bronstein

## **Graph Neural Networks**

- Input: graph-structured data
- Output: function that map graph to *learned representation*



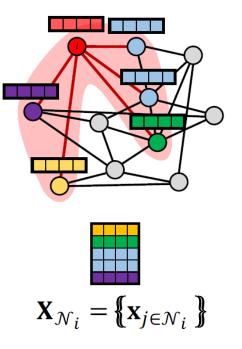
- How to learn representation?
- *Message passing mechanism* aggregates information from neighbors to update the representation

Message Passing mechanism

1. Node  $\mathbf{x}_i$  gathers messages from its neighbors  $\mathcal{N}(\mathbf{x}_i)$  $\mathbf{m}_i^{(l)} = AGGREGATE(\{\mathbf{h}_j^{(l-1)}: j \in \mathcal{N}(i)\})$ 

2. The representation of  $\mathbf{x}_i$  is updated

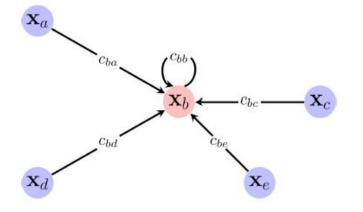
$$\mathbf{h}_{i}^{(l)} = \text{UPDATE}(\mathbf{h}_{i}^{(l-1)}, \mathbf{m}_{i}^{(l)})$$



### **Graph Neural Networks**

- Graph Convolutional Network (GCN)
- Node features aggregate using  $\hat{\mathbf{A}} = \mathbf{A} + \mathbf{I}$

$$\mathbf{H}^{(l+1)} = ReLU(\widehat{\mathbf{A}}\mathbf{H}^{(l)}\mathbf{W}^{(l)})$$



• Aggregation from more distant nodes:

Iteratively performing the message-passing and update node features (3-layer GCN)

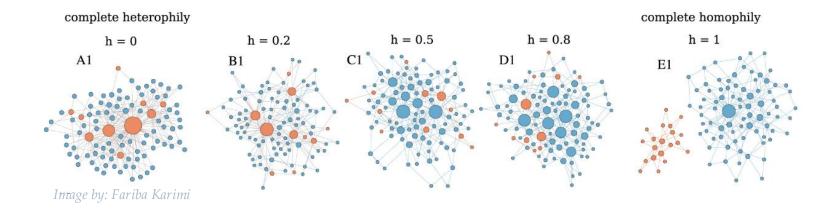
$$\mathbf{H}^{(4)} = ReLU(\widehat{\mathbf{A}} ReLU(\widehat{\mathbf{A}} ReLU(\widehat{\mathbf{A}} X \mathbf{W}^{(1)}) \mathbf{W}^{(2)}) \mathbf{W}^{(3)})$$

#### Useful for **homophilic** graphs !

Questions:

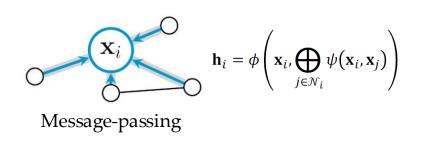
- 1- How to apply GCN on heterophilic graphs?
- 2- How to have long-distance communication between nodes with GCN?

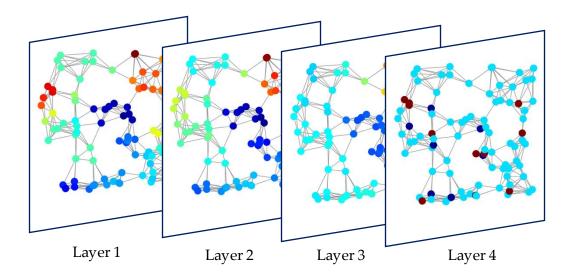
GCN with higher-order layers The models is prone to *oversmoothing* 



### Challenge

- Message-passing based approaches are prone to *oversmoothing*
- GNNs have **structural limitations** to apply on *large-scale* or *heterophilic* graphs!





### Our Goals:

- Avoid structural limitations of the message-passing frameworks
- Facilitate information propagation by using the diffusion equation
- Ensure long-distance communication between nodes

#### TIDE: Time Derivative Diffusion for Deep Learning on Graphs

TIDE: Time Derivative Diffusion for Deep Learning on Graphs

Maysam Behmanesh<sup>\*1</sup> Maximilian Krahn<sup>\*12</sup> Maks Ovsjanikov<sup>1</sup>

#### Abstract

A prominent paradigm for graph neural networks is based on the message-passing framework. In this framework, information communication is realized only between neighboring nodes. The challenge of approaches that use this paradigm is to ensure efficient and accurate long-distance communication between nodes, as deep convolutional networks are prone to oversmoothing. In this paper, we present a novel method based on time derivative graph diffusion (TIDE) to overcome these structural limitations of the message-passing framework. Our approach allows for optimizing the spatial extent of diffusion across various tasks and network channels, thus enabling medium and long-distance communication efficiently. Furthermore, we show that our architecture design also enables local message-passing and thus inherits from the capabilities of local message-passing approaches. We show that on both widely used graph benchmarks and synthetic mesh and graph datasets, the proposed framework outperforms state-of-the-art methods by a significant margin.+

#### 1. Introduction

Designing efficient and scalable architectures for learning on graphs is a central problem in machine learning with applications in a broad range of disciplines, including data (see, e.g., (Zhou et al., 2020; Wu et al., 2020) for recent surveys), ranging from spectral methods, spatial or convolutional designs, recurrent graph neural networks, or graph auto-encoders as well as many other hybrid techniques. A particularly prominent and widely-used category of approaches is given by the convolutional graph neural networks, and especially those based on message-passing, following the design introduced in (Kipf & Welling, 2017) and extended significantly in many follow-up works, e.g., (Li et al., 2018b; Zhuang & Ma, 2018; Chamberlain et al., 2021b; Thorpe et al., 2021).

The key strengths of convolutional graph neural networks, as introduced in (Kipf & Welling, 2017), include their simplicity and computational efficiency, their ability to be composed with other neural networks as well as their ability to generalize across different graphs (i.e., learning weights that could be applied on unseen graphs). As a result, the original GCN approach (Kipf & Welling, 2017) is still highly effective and is widely used in many applications.

Nevertheless, a prominent limitation of message-passing approaches, such as GCN and related methods is *oversmoothing*, which implies that such networks tend to be difficult to train beyond a small number of layers (Oono & Suzuki, 2019). Furthermore, since typical message-passing operators only ensure communication between nodes within a 1-hop neighborhood, this means that message-passing approaches can hinder *long-distance information propagation*, which can limit their utility in scenarios, where such longrange communication is important.







Maysam Behmanesh

nesh Maximilian Krahn

Maks Ovsjanikov

#### https://github.com/maysambehmanesh/TIDE

"TIDE: Time Derivative Diffusion for Deep Learning on Graphs," M. Behmanesh, M. Krahn, and M. Ovsjanikov, ICML, 2023

### Key idea: ensure information propagation using the *diffusion equation*

In the continuous setting, the diffusion process is described as the solution of *the heat equation* 

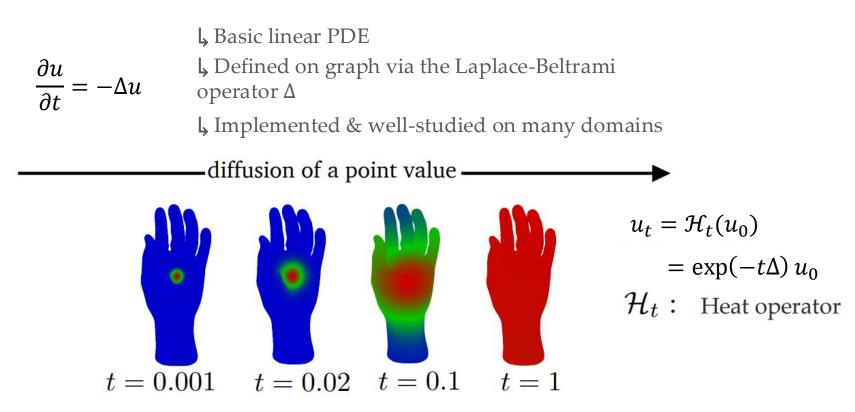


Image from GDL-course, Ovsjanikov

#### **Time-derivative diffusion**

#### Main goal:

Combine the local accuracy with the global information propagation (without oversmoothing)

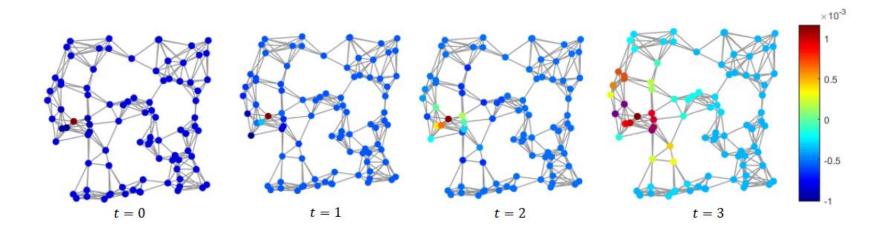
We propose time-derivative diffusion as a communication mechanism:

$$\frac{\partial u}{\partial t} = -\Delta u \quad \rightarrow \quad u_t = \mathcal{H}_t(u_0) = \exp(-t\mathbf{L}) u_0 \quad \rightarrow \quad -\frac{\partial u_t}{\partial t} = \mathbf{L}u_t = \mathbf{L}\exp(-t\mathbf{L}) u_0$$

TIDE combines local accuracy with global information propagation by:

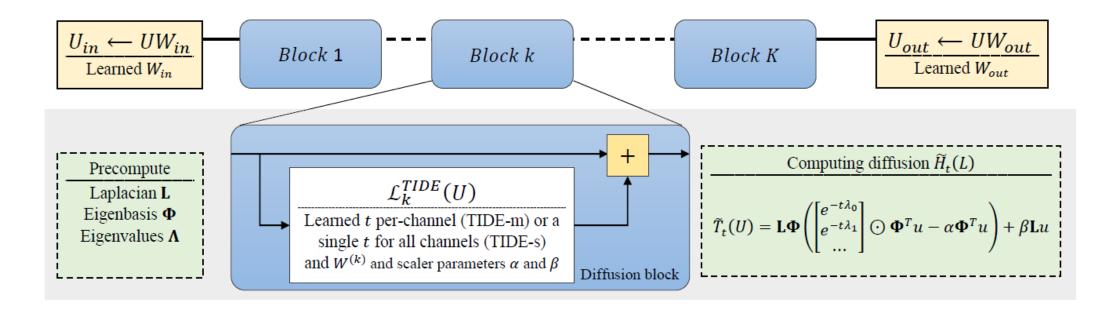
$$\mathcal{L}_{k}^{TIDE}(\mathbf{U}) = \sigma(T_{t}(\mathbf{U})\mathbf{W}^{(k)}) = \sigma(\mathbf{L}\exp(-t_{k}\mathbf{L})\mathbf{U}\mathbf{W}^{k})$$

Key idea: Using *learnable time diffusion* which allows information propagation on the graph



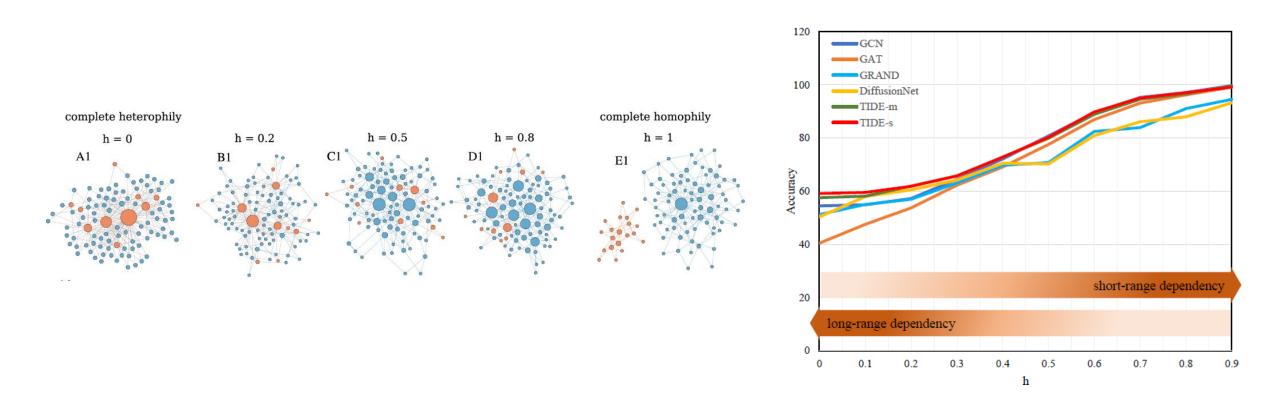
L, small t, the support is local and the output is concentrated at the center node L, larger t, the spatial support increases, facilitating distant communication between nodes

### **TIDE** Architecture



Two variants of TIDE: **TIDE-m:** learn *t* per-channel **TIDE-s:** learn a single *t* for all channels

### **Results**: Long Range Communication



### **Results:** Node Classification

Scale	Dataset	#Nodes	#Edges	#Feature	#Class	#CC	h%	Avg. N.D.	Diameter
C	Cora	2,708	5,429	1,433	7	78	80.4	4.08	19
Small	Citeseer	3,327	4,732	3,703	6	438	73.5	3.47	28
	PubMed	19,717	44,324	500	3	1	80.2	4.5	18
Mallin	CoauthorCs	18,333	81,894	6,805	15	1	80	8.93	24
Medium	Computers	13,381	245,778	767	10	314	77.7	36.74	10
	Photos	7,487	119,043	745	8	136	82.7	31.8	11
	ogbn-arxiv	169,343	1,166,243	128	40	1	65.4	13.67	23

**#CC**: Number of connected components, **h**%: Homophily rate, **Avg. N.D**: Average node degrees

Model	Cora	Citeseer	Pubmed	CoauthorCS	Computer	Photo	Ogbn-arxiv
GCN (Kipf & Welling, 2017)	83.30±0.36	68.23±0.91	76.78±0.31	90.17±0.50	81.01±0.65	91.71±0.67	65.91±0.12
GAT (Veličković et al., 2017)	81.83±0.42	69.19±0.53	75.49±0.43	90.15±0.35	80.25±0.52	91.57±0.41	54.23±0.22
GRAND (Chamberlain et al., 2021b)	80.71±0.86	68.06±0.18	74.61±0.25	90.59±0.21	72.96±0.49	84.17±0.34	59.29±0.12
GCNII (Chen et al., 2020)	$79.94 \pm 1.11$	70.27±0.32	76.59±0.7	84.27±0.80	32.63±8.6	57.41±3.6	49.87±0.37
ACM (Luan et al., 2022)	81.83±0.12	69.03±0.02	73.3±0.63	91.50±0.13	77±0.65	92.42±0.29	66.23±0.42
DiffusionNet (Sharp et al., 2022)	80.96±0.50	70.00±0.91	73.09±0.15	89.52±0.22	74.72±0.66	87.17±0.26	54.79±0.16
TIDE-m	84.47±0.43	70.32±0.68	77.59±0.04	89.86±0.30	82.11±0.03	91.33±0.47	67.86±1.10
TIDE-s	84.31±0.36	70.24±0.80	<u>77.24±0.62</u>	<u>90.21±0.12</u>	83.01±0.02	<u>92.06±0.51</u>	68.43±0.35

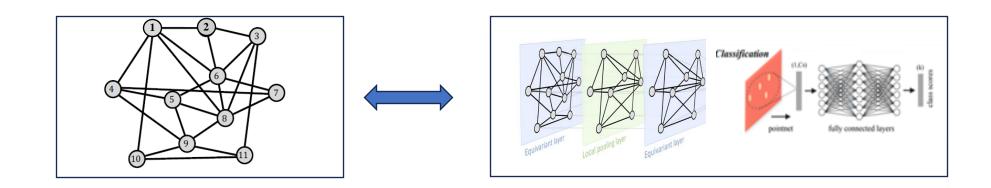
### **Results:**

### Node Classification on graphs with different homophily rates

Graph	#Nodes	#Edges	#Node featues	#Class	Class type	h
Chameleon	2,277	36,101	2,325	5	Wiki pages	0.23
Actor	7,600	29,926	931	5	Actors in movies	0.22
Cornell	183	295	1,703	5	Web pages	0.3
Texas	183	309	1,703	5	Web pages	0.11
Wisconsin	251	499	1,703	5	Web pages	0.21
Genius	421,961	984,979	12	2	marked act.	0.618
Twitch-gamers	168,114	6,797,557	7	2	mature content	0.545
Snap-patents	2,923,922	13,975,788	269	5	time granted	0.073

Model	Chameleon	Actor	Cornell	Texas	Wisconsin	Genius	Twitch-gamer	Snap-patents
GCN	45.18±0.62	29.38±0.5	43.24±1.3	63.51±1.9	54.92±9.7	80.87±0.13	60.60±0.19	36.84±0.37
GAT	44.96±6.2	28.88±1.0	54.05±1.1	62.16±0.08	55.88±1.4	79.83±0.23	53.08±0.16	38.76±0.75
GRAND	50.33±0.47	35.00±0.28	55.41±1.9	67.62±1.9	64.86±1.3	82.47±0.08	59.85±0.03	38.89±0.42
DiffusionNet	53.84±1.1	34.44±0.33	56.76±0.6	62.16±0.0	62.78±2.8	82.59±0.12	55.72±1.6	30.69±0.014
TIDE-m	<u>52.08±1.1</u>	<u>36.18±0.47</u>	<u>58.11±1.9</u>	<u>64.86±1.5</u>	69.61±1.4	83.03±0.06	60.81±0.04	40.56±1.7
TIDE-s	51.75±0.47	36.64±0.47	59.46±1.9	63.81±1.9	<u>68.63±1.4</u>	83.01±0.06	60.40±0.13	40.75±0.58

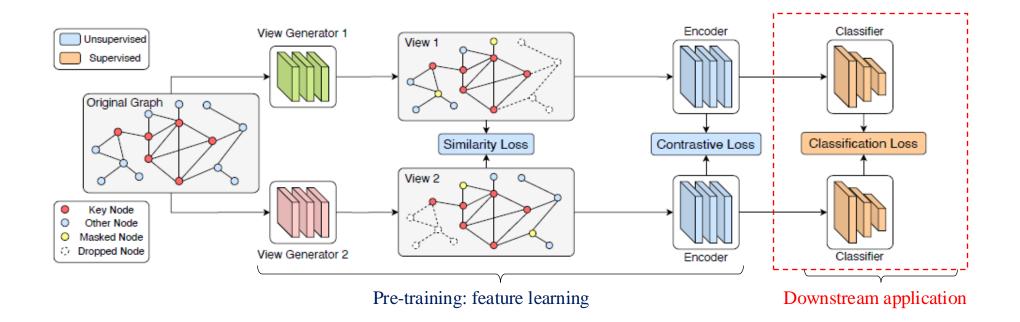
- 1. GNNs are Limited to *data-rich applications*, not useful in generic tasks!
- 2. Lack of *generalizable* (transfer) learning on graph



### **Research direction**

#### Typical representation learning pipeline

**Contrastive Learning:** powerful feature learning without labels.

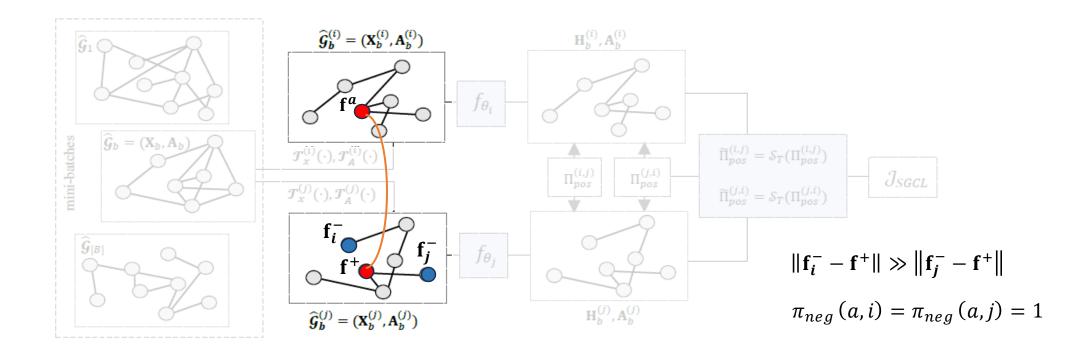


How to learn **informative features** on unlabeled graph (ideally, useful in **downstream applications**) ?

### Challenge

### Major challenge:

• GCL allocates negative pairs *uniformly*, regardless of their proximity to the true positive.



### **Our Goal:**

• Integration proximity information in the contrastive loss.

We expect:

 $0 < \pi_{neg}(a,j) < \pi_{neg}(a,i) < 1$ 

#### Smoothed Graph Contrastive Learning via Seamless Proximity Integration

Integra	ation
Maysam Behmanesh	Maks Ovsjanikov
LIX, École polytechnique, IP Paris behmanesh@lix.polytechnique.fr	LIX, École polytechnique, IP Paris maks@lix.polytechnique.fr
Abstr	act
GCL approaches incorporate negative san resulting in the equal treatment of negativ to the true positive. In this paper, we pr Learning model (SGCL), which leverages graphs to inject proximity information a in the contrastive loss, thus significantly 1 proposed SGCL adjusts the penaltics ass loss by incorporating three distinct smooth aware positives and negatives. To enhance proposed framework incorporates a grap tions the given graphs into multiple subgesparate batches. Through extensive expen-	ve nodes, regardless of their proximity resent a Smoothed Graph Contrastive st be geometric structure of augmented ssociated with positive/negative pairs regularizing the learning process. The sociated with node pairs in contrastive sing techniques that result in proximity- e scalability for large-scale graphs, the h batch-generating strategy that parti- raphs, facilitating efficient training in

Graph Neural Networks (GNNs) [1–3] have developed rapidly by providing powerful frameworks for the analysis of graph-structured data. A significant portion of GNNs primarily focus on (semi-) supervised learning, which requires access to abundant labeled data [2, 4, 5]. However, labeling graphs is challenging because they often represent specialized concepts within domains like biology.

Graph Contrastive Learning (GCL), as a new paradigm of Self-Supervised Learning (SSL) [6] in the graph domain, has emerged to address the challenge of learning meaningful representations from graph-structured data [7, 8]. They leverage the principles of self-supervised learning and contrastive loss [9] to form a simplified representation of graph-structured data without relying on supervised data.

In a typical GCL approach, several graph views are generated through stochastic augmentations of the input graph. Subsequently, representations are learned by comparing congruent representations





Maysam Behmanesh

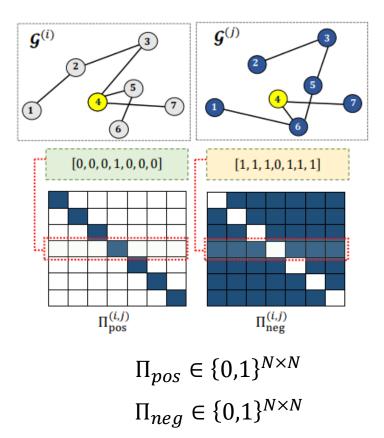
Maks Ovsjanikov

#### https://github.com/maysambehmanesh/SGCL

"Graphs Smoothed Graph Contrastive Learning via Seamless Proximity Integration," M. Behmanesh, and M. Ovsjanikov, LoG, 2024

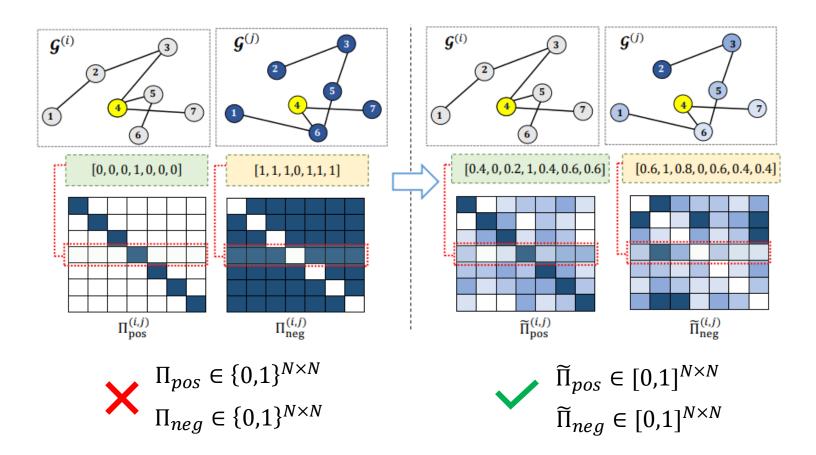
### **Our intuition:**

Going beyond simple binary categorization of positive and negative points



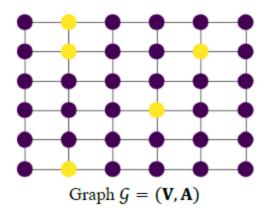
### **Our intuition:**

Going beyond simple binary categorization of positive and negative points



### Question:

How can proximity information be effectively incorporated into contrastive loss?

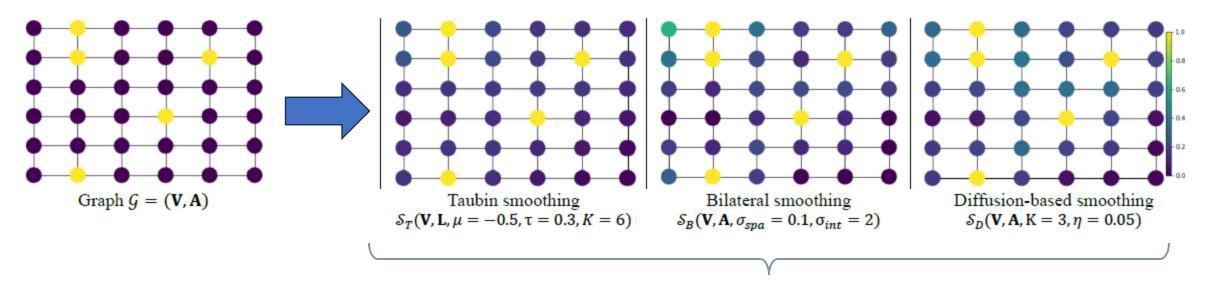


**Input**: binary matrix  $\Pi \in \{0,1\}^{N \times N}$ 

### **Our Intuition:**

Applying a smoothing approach for graph

Smoothing involves iteratively updating node values based on the values of their neighboring nodes



**Input**: binary matrix  $\Pi \in \{0,1\}^{N \times N}$ 

**Output:** smooth matrix  $\widetilde{\Pi} \in [0,1]^{N \times N}$ 

### SGCL

### **Smoothing approaches:**

Smoothing involves iteratively updating node values based on the values of their neighboring nodes

### 1. Taubin smoothing

$$\mathbf{V}^{(k+1)} = (\mathbf{I} + \tau \mathbf{L}) \left( (\mathbf{I} + \mu \mathbf{L}) \mathbf{V}^{(k)} \right), \quad \mu < 0, \tau > 0, \text{ and } \mu < -\tau$$

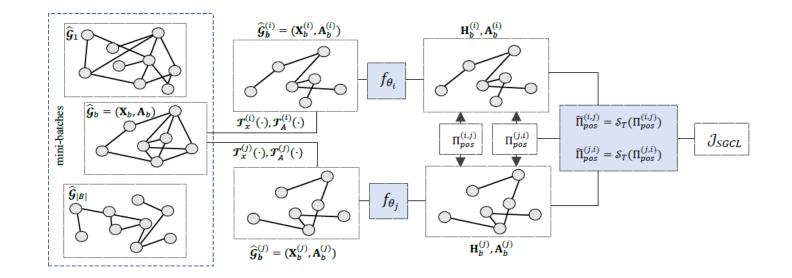
### 2. Bilateral smoothing

$$\widetilde{\mathbf{v}}_{i} = \frac{\sum_{j \in \mathcal{N}_{k}(i)} w(i,j) \mathbf{v}_{j}}{\sum_{j \in \mathcal{N}_{k}(i)} w(i,j)} , \text{ where } w(i,j) = \exp\left(-\frac{d_{spa}(i,j)}{2\sigma_{spa}^{2}} - \frac{d_{int}(i,j)}{2\sigma_{int}^{2}}\right)$$

### 3. Diffusion-based smoothing

$$\mathbf{v}_i^{(k+1)} = \mathbf{v}_i^{(k)} + \eta \ \overline{\mathbf{v}}_i^{(k)}$$
, where  $\overline{\mathbf{v}}_i^{(k)} = \sum_{j \in \mathcal{N}(i)} \mathbf{v}_j$ 

### SGCL – Architecture



**Contrastive Loss:** 

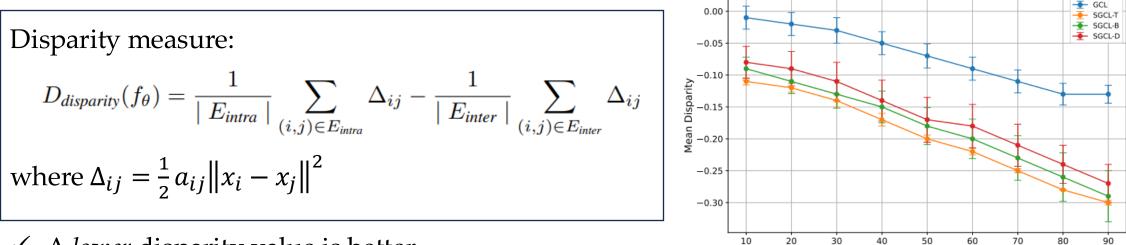
$$\mathcal{L}_{SGCL}^{(i,j)} = \| \tilde{\Pi}_{pos}^{(i,j)} \odot (\mathbf{1} - \mathbf{C}^{(i,j)}) \|_F^2 + \lambda \| \tilde{\Pi}_{neg}^{(i,j)} \odot \mathbf{C}^{(i,j)} \|_F^2$$

$$\mathbf{C}^{(i,j)}$$
: normalized cosine similarity between the embeddings

### SGCL

### Feature space analysis

Dirichlet energy  $E(\mathbf{X}) = \mathbf{X}^T \mathbf{L} \mathbf{X} = \frac{1}{2} \sum_{i,j} a_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2$  measures the *smoothness* of the signal **X** over the graph



✓ A *lower* disparity value is better

Model	Cora	Citeseer	Pubmed	CoauthorCS	Computers	Photo	ogbn-arxiv
GCL (GRACE)	$0.66 \pm 0.05$	$0.63 \pm 0.02$	$0.51 \pm 0.03$	$0.58 \pm 0.02$	$0.64 \pm 0.03$	$0.65 \pm 0.03$	0.56±0.03
SGCL-T	$0.49 \pm 0.05$	$0.58 \pm 0.05$	$0.50 \pm 0.01$	$0.53 \pm 0.03$	$0.56 \pm 0.01$	$0.60 \pm 0.05$	$0.51 \pm 0.02$
SGCL-B	$0.63 \pm 0.03$	0.51±0.05	$0.49 \pm 0.02$	$0.52 \pm 0.01$	$0.52 \pm 0.05$	$0.59 \pm 0.02$	$0.48 \pm 0.05$
SGCL-D	$0.47 \pm 0.02$	$0.54 \pm 0.03$	0.49±0.01	0.46±0.05	$0.55 \pm 0.05$	0.43±0.03	0.45±0.04

h (%)

### **Results**: Node Classification

Model	Cora	Citeseer	Pubmed	CoauthorCS	Computers	Photo
DGI [10]	82.3±0.6	71.8±0.7	76.8±0.6	92.15±0.63	83.95±0.47	91.61±0.22
GRACE [11]	83.3±0.4	72.1±0.5	73.63±0.20	91.12±0.20	89.53±0.35	92.78±0.45
MVGRL [12]	83.11±0.12	73.3±0.5	84.27±0.04	92.11±0.12	87.52±0.11	91.74±0.07
BGRL [21]	83.77± 0.57	73.07±0.06	84.62±0.35	93.31±0.13	90.34±0.19	93.17±0.3
G-BT [22]	83.63±0.44	72.95±0.17	84.52±0.12	92.95±0.17	88.14±0.33	92.63±0.44
CGRA [23]	83.8±0.4	69.23±1.19	82.8±0.4	92.8±0.5	90.5±0.4	92.4±0.2
GRLC [24]	83.5±0.5	72.6±0.6	82.1±0.4	90.36±0.27	88.54±0.23	92.3±0.5
ProGCL-weight [25]	81.91±0.12	69.24±0.21	84.89±0.04	93.51±0.06	89.28±0.15	93.30±0.09
ProGCL-mix [25]	83.71±0.04	68.38±0.3	84.64±0.03	93.67±0.12	89.55±0.16	93.64±0.13
GraphMAE2 [26]	84.5±.0.6	73.4±0.3	81.4±0.5	-	_	_
AUGCL [27]	_	_	_	-	88.94±0.44	93.43±0.32
GREET [28]	83.81±0.87	73.08±0.84	80.29±1.00	94.65±0.18	87.94±0.35	92.85±0.31
SGCL-T	84.33±0.45	74.94±0.79	84.25±0.35	92.25±0.15	87.21±0.42	93.12±0.7
SGCL-B	84.78±0.3	74.30±1.4	84.1±0.25	92.33±0.4	89.75±0.8	93.72±0.12
SGCL-D	84.17±0.43	75.72±0.59	85.12±0.3	92.14±0.26	86.11±0.3	92.87±0.6

### **Results**: Node Classification (large scale graphs)

Scale	Dataset	#Nodes	#Edges	#Feature	#Class	#CC	h%	Avg. N.D.	Diameter
Large	ogbn-arxiv	169,343	1,166,243	128	40	1	65.4	13.67	23
	ogbn-products	2,449,029	61,859,140	100	47	52,658	80.8	51.54	27
	ogbn-proteins	132,534	39,561,252	8	94	1	91	597	9

Model	ogbn-arxiv	ogbn-products	ogbn-proteins
DGI [10]	67.07±0.5	68.68±0.6	94.11±0.1
GRACE [11]	67.92±0.4	72.10±0.7	94.11±0.2
MVGRL [12]	60.68±0.5	69.90±0.9	93.87±0.3
BGRL [21]	63.88±0.2	66.23±0.5	92.94±0.3
GBT [22]	69.05±0.3	65.74±0.4	94.07±0.3
GraphMAE2 [26]	68.95±0.4	74.32±0.5	_
SGCL-T	70.89±0.2	75.97±0.1	94.64±0.2
SGCL-B	70.34±0.4	74.33±0.4	93.55±0.2
SGCL-D	70.52±0.3	74.15±0.2	93.19±0.1

SGCL

Dataset	#Graph	Avg. node	Avg. edge	#Features	#Class
MUTAG	188	17.9	39.6	7	2
PTC-MR	344	14.29	14.69	19	2
IMDB-Binary	1,000	19.8	193.1	1	2
PROTEINS	1,113	39.1	145.6	3	2
ENZYMES	600	32.63	124.3	3	6

### **Results**: Graph Classification

Model	IMDB-Binary	PTC-MR	MUTAG	PROTEINS	ENZYMES
InfoGraph [29]	73.0±0.9	61.7±1.4	89.0±1.1	74.4±0.3	50.2±1.4
GraphCL [30]	71.1±0.4	63.6±1.8	86.8±1.3	74.4±0.5	55.1±1.6
MVGRL [12]	74.2±0.7	62.5±1.7	89.7±1.1	71.5±0.3	48.3±1.2
AD-GCL [31]	71.5±1.0	$61.2 \pm 1.4$	86.8±1.3	75.0±0.5	42.6±1.1
BGRL [21]	72.8±0.5	57.4±0.9	86.0±1.8	77.4±2.4	50.7±9.0
LaGraph [32]	73.7±0.9	$60.8 \pm 1.1$	<u>90.2±1.1</u>	75.2±0.4	40.9±1.7
ProGCL-mix [25]	71.6±0.6	_	88.7±1.4	74.5±0.4	_
CGRA [23]	75.6±0.5	65.7±1.8	91.1±2.5	76.2±0.6	61.1±0.9
AUGCL [27]	72.4±0.8	_	89.2±1.0	75.7±0.4	_
SGCL-T	75.2±2.8	<u>64.0±1.6</u>	89.0±2.3	79.4±1.9	65.3±3.6
SGCL-B	73.2±3.7	62.5±1.8	87.0±2.8	81.6±2.3	63.7±1.6
SGCL-D	75.8±1.9	62.6±1.4	86.0±2.6	<u>81.5±2.3</u>	<u>64.3±2.2</u>

1- Most message-passing-based GNNs are prone to *oversmoothing* 

L, TIDE facilitates information propagation using the diffusion equation L, TIDE ensures long-distance communication between nodes

2- Conventional GCL frameworks allocates negative pairs *uniformly*, regardless of their proximity

L SGCL incorporates the geometric structure of graph into a smoothed contrastive loss L SGCL intuitively consider proximity information in assigning positive and negative pairs Thank You For Your Attention

# Questions?



#### Acknowledgements:

Part of this work are supported by the ERC Starting Grant No. 758800 (EXPROTEA) and ERC Consolidator Grant 101087347 (VEGA).